Çankaya University – ECE Department – ECE 587

Student Name : Duration : 2 hours
Student Number : Open book exam

Questions

1. (35 Points) For the $(n = 2,\ k = 1)$ convolutional encoder shown below, find the output by hand and using Matlab, if the input message sequence is $\mathbf{x} = [1,\ 0,\ 1,\ 1,\ 0,\ 0,\ 1]$. Make sure that in the two cases, the output (code word) is the same. Draw the related state transition diagram and trellis diagram. Using the trellis diagram, show by hand tracing that the code word will give back the input message. Confirm this using Matlab.

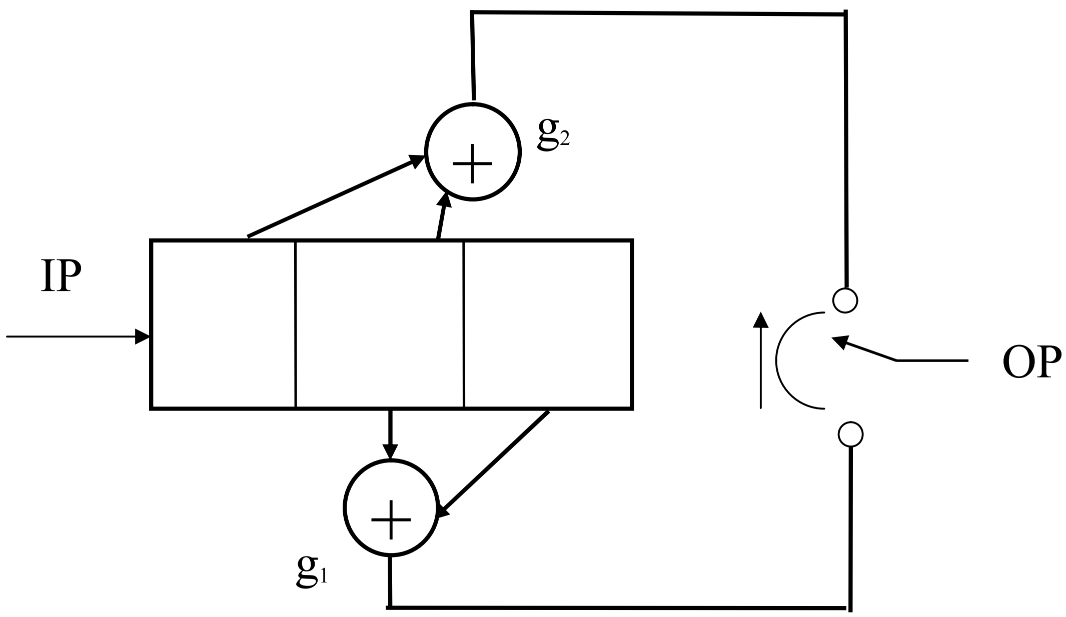

Fig. 1.1 Convolution encoder for Q1.

Solution : We assume that the shift registers are initially loaded with zeros, then by hand tracing we get the following

Input	Contents of shift registers	$\mathbf{g}_1$	$\mathbf{g}_2$	$\mathbf{c}$ - Output
1	1 0 0	0	1	01
0	0 1 0	1	1	11
1	1 0 1	1	1	11
1	1 1 0	1	0	10
0	0 1 1	0	1	01
0	0 0 1	1	0	10
1	1 0 0	0	1	01

$$\mathbf{x}^t = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{1.1}$$

So the output of the convolutional encoder shown in Fig. 1.1 is

$$\begin{aligned} \mathbf{x} &= [1, \quad 0, \quad 1, \quad 1, \quad 0, \quad 0, \quad 1] \\ & \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ \mathbf{c} &= [\overbrace{0,\ 1}^{1}\ \overbrace{1,\ 1}^{0}\ \overbrace{1,\ 1}^{1}\ \overbrace{1,\ 0}^{1}\ \overbrace{0,\ 1}^{0}\ \overbrace{1,\ 0}^{0}\ \overbrace{0,\ 1}^{1}] \end{aligned} \tag{1.2}$$

By recognizing that in Fig. 1.1 and using (4.5) and (4.6) of ECE 587_Notes on Codes

$$\mathbf{g}_1 = [0,\ 1,\ 1] \qquad \mathbf{g}_2 = [1,\ 1,\ 0] \tag{1.3}$$

We write for the poly2trellis function of Matlab as follows

$$\begin{aligned} &\text{Constraint length (number of shift registers) - } L \quad \mathbf{g}_1 \quad \mathbf{g}_2 \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \downarrow \quad \downarrow \quad \downarrow \\ &\qquad\qquad\qquad \text{t} = \text{poly2trellis}(3, \quad [3, \quad 6]) \end{aligned} \tag{1.4}$$

Subsequently running Conv12_Exp2.m with the input in (1.2) and the ploys2trellis function of Matlab given in (1.4), we get the same output as written on the final line of (1.2).

The state transition diagram of the convolutional encoder of Fig. 1.1 is given in Fig. 1.2.

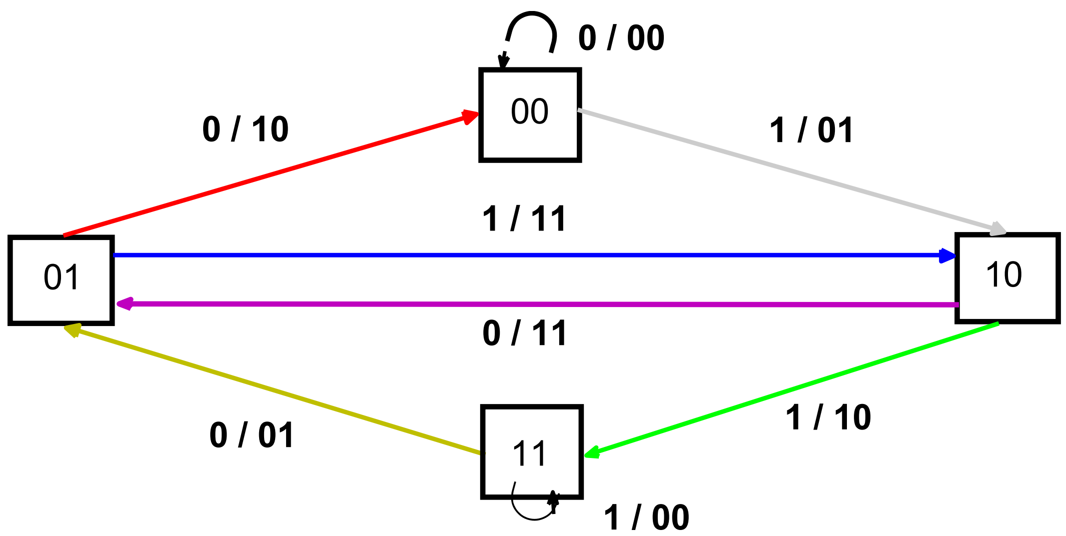

Fig. 1.2 State transition diagram of the convolutional encoder of Fig. 1.1.

The decoding traced on the trellis diagram is displayed in Fig. 1.3.

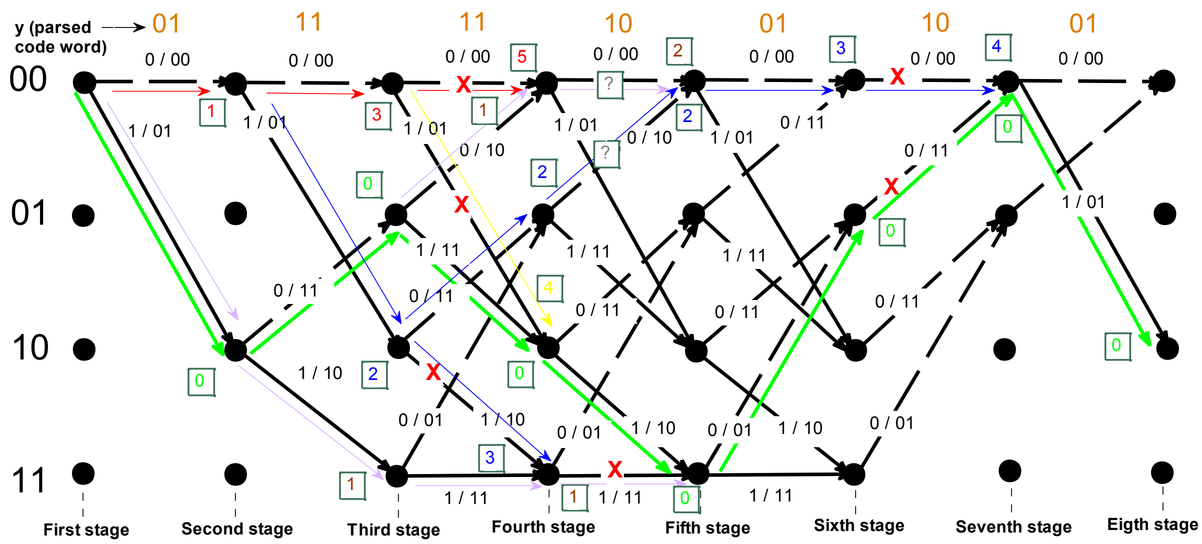

Fig. 1.3 The decoding traced on the trellis diagram for the convolutional encoder of Fig. 1.1.

Note that in Fig. 1.3, the path that leads to the input that was originally input fed to the convolutional encoder of Fig. 1.1 is marked in green. Further note that in Fig. 1.3, not all path searches are covered. Using the Matlab file, Conv12_Exp2.m, we confirm the result obtained in Fig. 1.3.

2. (35 Points) By observing that

$$p^6+1=\left(p^2+1\right)\left(p^2+p+1\right)\left(p^2+p+1\right) \tag{2.1}$$

Construct a $\left(n=6,\ k=4\right)$ cyclic code. Test its cyclic and linearity property. Also evaluate the Hamming distance, weights of all code words, the minimum distance and the minimum weight.

Solution : There is one polynomial, satisfying the condition $n-k=2$, namely $g\left(p\right)=p^2+1$. Hence from subsection 3.3 of ECE 587_Notes on Codes, we obtain

Take p^{n-1}, i. e., p^5 divide it by $g\left(p\right)=p^2+1$, remainder : p

$$p^4 \bmod p^2+1=1 \qquad \leftarrow \text{remainder}$$
$$p^3 \bmod p^2+1=p \qquad \leftarrow \text{remainder}$$
$$p^2 \bmod p^2+1=1 \qquad \leftarrow \text{remainder}$$

Sample computation

$$\begin{array}{l|l} p^5 & p^2+1 \\ \underline{p^5+p^3} & \overline{p^3+p} \\ 0+p^3 & \\ \underline{\quad p^3+p} & \\ \quad\quad p \leftarrow \text{remainder} & \end{array} \tag{2.1}$$

Then using (3.14) of ECE 587_Notes on Codes, we find the generator matrix, $\mathbf{G}$ for this cyclic code in systematic form as follows

$$\mathbf{G}=\left[\mathbf{I}_k \middle| \mathbf{P}\right]=\left[\begin{array}{cccc|cc} 1&0&0&0&1&0 \\ 0&1&0&0&0&1 \\ 0&0&1&0&1&0 \\ 0&0&0&1&0&1 \end{array}\right],\quad \mathbf{I}_k=\begin{bmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{bmatrix},\quad \mathbf{P}=\begin{array}{c} p^1\ \ p^0 \\ \begin{bmatrix} 1&0 \\ 0&1 \\ 1&0 \\ 0&1 \end{bmatrix} \end{array} \begin{array}{c} \leftarrow \ \text{remainder} \\ \Leftarrow \ \boxed{\begin{array}{c} p \\ 1 \\ p \\ 1 \end{array}} \end{array} \tag{2.2}$$

It is also possible to arrive at a generator matrix in nonsystematic manner as shown in (2.3)

$$\mathbf{G}=\begin{array}{c} \begin{array}{cccccc} p^5 & p^4 & p^3 & p^2 & p^1 & p^0 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \end{array} \\ \begin{bmatrix} 1&0&1&0&0&0 \\ 0&1&0&1&0&0 \\ 0&0&1&0&1&0 \\ 0&0&0&1&0&1 \end{bmatrix} \end{array} \begin{array}{l} \leftarrow p^3 g\left(p\right)=p^5+p^3 \\ \leftarrow p^2 g\left(p\right)=p^4+p^2 \\ \leftarrow p^1 g\left(p\right)=p^3+p \\ \leftarrow p^0 g\left(p\right)=p^2+1 \end{array} \tag{2.3}$$

In Table 2.1, we construct the code words using $\mathbf{G}$ in (2.3)

Inserting $\mathbf{G}$ of (2.2) in Cyccodesys_ECE587.m, we get the following code table

i	$\mathbf{x}_i$	$X(p)$	$c(p)=X(p)g(p)$	$\mathbf{c}_i$	$\mathbf{w}_i$
1	0000	0	0	000000	0
2	0001	1	p^2+1	000101	2
3	0010	p	p^3+p	001010	2
4	0100	p^2	p^4+p^2	010100	2
5	1000	p^3	p^5+p^3	101000	2
6	0011	$p+1$	p^3+p^2+p+1	001111	4
7	0110	p^2+p	$p^4+p^3+p^2+p$	011110	4
8	1100	p^3+p^2	$p^5+p^4+p^3+p^2$	111100	4
9	1001	p^3+1	$p^5+p^3+p^2+1$	101101	4
10	0101	p^2+1	p^4+1	010001	2
11	1010	p^3+p	p^5+p	100010	2
12	0111	p^2+p+1	p^4+p^3+p+1	011011	4
13	1110	p^3+p^2+p	$p^5+p^4+p^2+p$	110110	4
14	1101	p^3+p^2+1	$p^5+p^4+p^3+1$	111001	4
15	1011	p^3+p+1	p^5+p^2+p+1	100111	4
16	1111	p^3+p^2+p+1	p^5+p^4+p+1	110011	4

Table 2.1 List of code words for the cyclic code of $(n=6,\ k=4)$ in nonsystematic manner.

Tests conducted with Cyccodesys_ECE587.m shows that Table 2.1 satisfies linearity and cyclic property.

From Table 2.1, it is easy to see that $w_{\min}=d_{\min}=2$

3. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones justify your answer.

a) Linear codes have cyclic property : False, linear codes only have the linearity property, not the cyclic property. Linearity implies that the linear combination of two code words (in modulo-2 sense) is another code word in the same code.

b) TCM involves convolutional coding : True, TCM is based on convolutional encoding as explained in section 5 of ECE 587_Notes on Codes.

c) The purpose of coding is to make the symbol duration longer, thus enabling bandwidth reduction : False, coding reduces bit duration, thus increasing bandwidth requirement.

d) In convolutional coding, state transition diagram shows the output against input : In state transition diagram, we can only see the output from one state to the other upon an input. It is the trellis diagram that shows the output against an input along time axis.

e) Decoding of convolutional coding is achieved by trellis diagram : True, as explained in 4.3 of ECE 587_Notes on Codes.

f) Parity matrix is obtained by inverting the generator matrix : False, the way to obtain the parity matrix is to use the relations given in (2.18) and (2.19) of ECE 587_Notes on Codes.