Çankaya University – ECE Department – ECE 587

Student Name : Duration : 2 hours
Student Number : Open book exam

Questions

1. (70 Points)The convolutional encoder shown below is intended to convert 4 PSK into 8 PSK TCM. Find the output by hand and using Matlab m file Conv12_Exp2.m, if the input message sequence is $\mathbf{x} = 1, 1, 0, 1, 0, 0, 1, 0$. Draw the related state trellis diagram. Organize the output (for all possible inputs) in the form of Tables similar to Table 5.1 of ECE 587_Notes on Codes. From there and the state trellis diagram, make an assignment of symbols according to partition rules of Ungerboeck, then draw the constellation diagram of 8 PSK TCM. Using the trellis diagram, place the signal vectors correctly in TCM_Pe8PSK.m which compares the probability of error performance of 8 PSK TCM against uncoded 4 PSK for the encoder configuration in Fig. 1.1 and constellation that you have derived. Use the model files TCM_Dec.mdl and TCM_Enc.mdl during these runs. All Matlab files are available on course webpage.

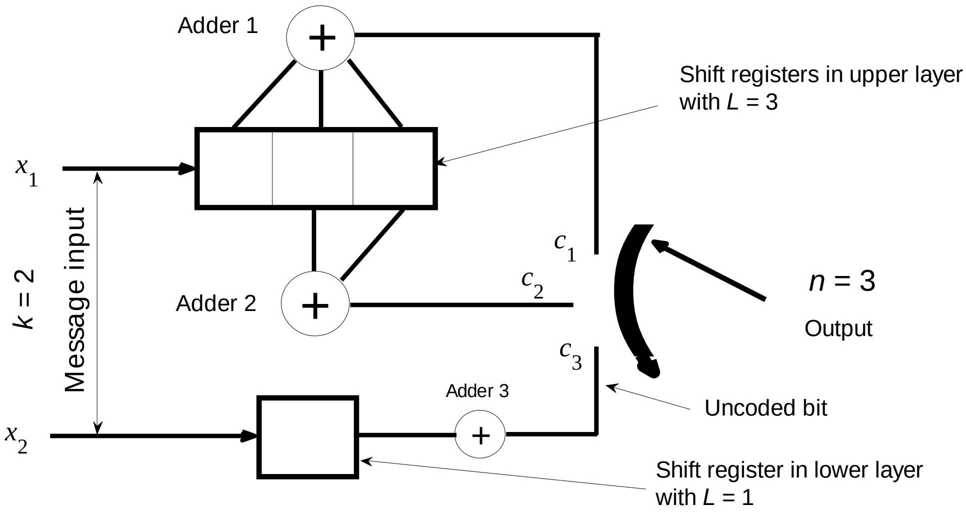

Fig. 1.1 Convolution encoder for Q1.

Solution : We assume that the shift registers are initially loaded with zeros, then by hand tracing we get the following

Input $\mathbf{x}^t$	Contents of shift registers in upper layer	Adder 1	Adder 2	Adder 3	$\mathbf{c}$ - Output $c_1\ c_2\ c_3$	
	0 0 0	0	0	0	0 0 0	(Prior to any input)
1 1	1 0 0	1	0	1	1 0 1	
0 1	0 1 0	1	1	1	1 1 1	(1.1)
0 0	0 0 1	1	1	0	1 1 0	
1 0	1 0 0	1	0	0	1 0 0	
$x_1\ x_1$						

So the output of the convolutional encoder shown in Fig. 1.1 is

$$\mathbf{x}=\begin{bmatrix}1,\ 1, & 0,\ 1, & 0,\ 0, & 1,\ 0\end{bmatrix}$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$
$$\scriptstyle 11 \qquad 01 \qquad 00 \qquad 10$$
$$\mathbf{c}=\ [1\,0\,1,\ 1\,1\,1,\ \ 1\,1\,0,\ \ 1\,0\,0] \qquad (1.2)$$

By considering Fig. 1.1 and using (4.5) and (4.6) of ECE 587_Notes on Codes

$$\text{Adder 1}=\begin{bmatrix}1,\ 1,\ 1\\0,\ 0,\ 0\end{bmatrix}=\begin{bmatrix}7\\0\end{bmatrix}\quad \text{Adder 2}=\begin{bmatrix}0,\ 1,\ 1\\0,\ 0,\ 0\end{bmatrix}=\begin{bmatrix}3\\0\end{bmatrix}\quad \text{Adder 3}=\begin{bmatrix}0,\ 0,\ 0\\0,\ 0,\ 1\end{bmatrix}=\begin{bmatrix}0\\1\end{bmatrix}\ \begin{bmatrix}\text{Upper layer}\\\text{Lower layer}\end{bmatrix} \qquad (1.3)$$

We write for the poly2trellis function of Matlab as follows

Constraint length (number of shift registers in upper and lower layers) - L ↓

Connection of Adder 1 to shift registers in upper layer ↓

Connection of Adder 3 to shift registers in upper layer ↓

$$\text{t} \ = \ \text{poly2trellis}\left([3,\ 1],\ \ [7,\ 3, \qquad 0;0\ 0\ 1]\right)$$

↑ Connection of Adder 2 to shift registers in upper layer

↑ Connection of Adder 3 to shift register in lower layer

(1.4)

Subsequently running Conv12_Exp2.m with the input in (1.2) and the ploys2trellis function of Matlab given in (1.4), we get the same output as written on the final line of (1.2).

Either by hand or by using Conv12_Exp2.m, we get the state trellis diagram drawn in Fig. 1.2. From there we derive Table 1.1 for originating states and Table 1.2 for terminating states.

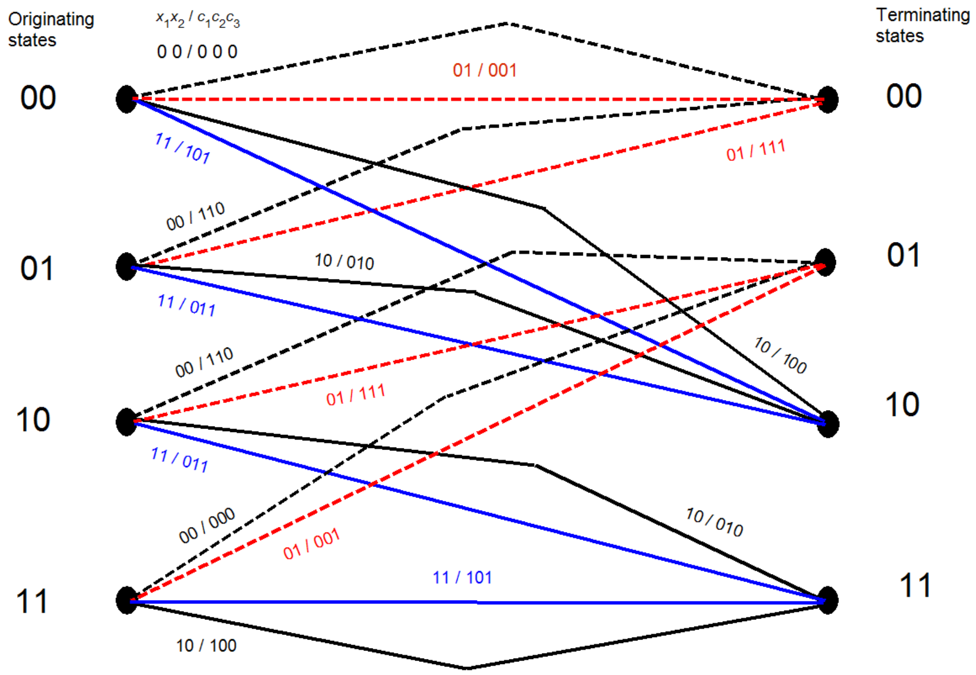

Fig. 1.2 State trellis diagram of the TCM encoder in Fig. 1.1.

Originating state	Input / Output symbol	**s** designation of output sysmbol	**B** designation	Parallel paths
00	00/000	$\mathbf{s}_1$	$\mathbf{B}_{1o}$	Parallel
00	01/001	$\mathbf{s}_2$	$\mathbf{B}_{1o}$	
00	10/100	$\mathbf{s}_5$	$\mathbf{B}_{1o}$	Parallel
00	11/101	$\mathbf{s}_6$	$\mathbf{B}_{1o}$	
01	00/110	$\mathbf{s}_7$	$\mathbf{B}_{2o}$	Parallel
01	01/111	$\mathbf{s}_8$	$\mathbf{B}_{2o}$	
01	10/010	$\mathbf{s}_3$	$\mathbf{B}_{2o}$	Parallel
01	11/011	$\mathbf{s}_4$	$\mathbf{B}_{2o}$	
10	00/110	$\mathbf{s}_7$	$\mathbf{B}_{2o}$	Parallel
10	01/111	$\mathbf{s}_8$	$\mathbf{B}_{2o}$	
10	10/010	$\mathbf{s}_3$	$\mathbf{B}_{2o}$	Parallel
10	11/011	$\mathbf{s}_4$	$\mathbf{B}_{2o}$	
11	00/000	$\mathbf{s}_1$	$\mathbf{B}_{1o}$	Parallel
11	01/001	$\mathbf{s}_2$	$\mathbf{B}_{1o}$	

11	10/100	$\mathbf{s}_5$	$\mathbf{B}_{1o}$	Parallel
11	11/101	$\mathbf{s}_6$	$\mathbf{B}_{1o}$	

Table 1.1 Organization of the originating state information in Fig. 1.2 in the form of a table.

Terminating state	Output symbol	$\mathbf{s}$ designation	B designation	Parallel paths
00	00/000	$\mathbf{s}_1$	$\mathbf{B}_{1t}$	Parallel
00	01/001	$\mathbf{s}_2$	$\mathbf{B}_{1t}$	
00	00/110	$\mathbf{s}_7$	$\mathbf{B}_{1t}$	Parallel
00	01/111	$\mathbf{s}_8$	$\mathbf{B}_{1t}$	
01	00/110	$\mathbf{s}_7$	$\mathbf{B}_{1t}$	Parallel
01	01/111	$\mathbf{s}_8$	$\mathbf{B}_{1t}$	
01	00/000	$\mathbf{s}_1$	$\mathbf{B}_{1t}$	Parallel
01	01/001	$\mathbf{s}_2$	$\mathbf{B}_{1t}$	
10	10/100	$\mathbf{s}_5$	$\mathbf{B}_{2t}$	Parallel
10	11/101	$\mathbf{s}_6$	$\mathbf{B}_{2t}$	
10	10/010	$\mathbf{s}_3$	$\mathbf{B}_{2t}$	Parallel
10	11/011	$\mathbf{s}_4$	$\mathbf{B}_{2t}$	
11	10/010	$\mathbf{s}_3$	$\mathbf{B}_{2t}$	Parallel
11	11/011	$\mathbf{s}_4$	$\mathbf{B}_{2t}$	
11	10/100	$\mathbf{s}_5$	$\mathbf{B}_{2t}$	Parallel
11	11/101	$\mathbf{s}_6$	$\mathbf{B}_{2t}$	

Table 1.2 Organization of the terminating state information in Fig. 1.2 in the form of a table.

From Table 1.1, we have the following $\mathbf{B}$ square designations.

$$\mathbf{B}_{1o} = (000,\ 001,\ 100,\ 101) \quad , \quad \mathbf{B}_{2o} = (010,\ 011,\ 110,\ 111)$$
$$\mathbf{B}_{1o} = (\ \mathbf{s}_1,\ \ \mathbf{s}_2,\ \ \mathbf{s}_5,\ \ \mathbf{s}_6) \quad , \quad \mathbf{B}_{2o} = (\ \mathbf{s}_3,\ \ \mathbf{s}_4,\ \ \mathbf{s}_7,\ \ \mathbf{s}_8) \tag{1.5}$$

Whereas, the square designations of Table 1.2, are the followings.

$$\mathbf{B}_{1t} = (000,\ 001,\ 110,\ 111) \quad , \quad \mathbf{B}_{2t} = (010,\ 011,\ 100,\ 101)$$
$$\mathbf{B}_{1t} = (\ \mathbf{s}_1,\ \ \mathbf{s}_2,\ \ \mathbf{s}_7,\ \ \mathbf{s}_8) \quad , \quad \mathbf{B}_{2t} = (\ \mathbf{s}_3,\ \ \mathbf{s}_4,\ \ \mathbf{s}_5,\ \ \mathbf{s}_6) \tag{1.6}$$

Comparing (1.5) to (1.6), we do not find the same $\mathbf{B}$ designations for originating and terminating cases. Recall that in Table 5.1 of ECE 587_Notes on Codes, the same $\mathbf{B}$ designations for originating and terminating cases cloud be made. From these observations, we deduce that it is not possible to apply partitioning rules of Ungerboeck to the encoder of

Fig. 1.1 and obtain desirable probability of error performance in 8 PSK TCM, compared with uncoded 4 PSK. To illustrate this, we use Table 1.1, and 1.2 or (1.5 and 1.6) and assign the symbols of 8 PSK TCM to two **B** squares. Running TCM_Pe8PSK.m with the constellation of Fig. 1.3, we get the curves in Figs. 1.3 and 1.4. As seen from these figures, the performance of 8 PSK TCM based on the encoder configuration of Fig. 1.1 is no so favorable as that found in lecture notes entitled, "ECE 587_Notes on Codes".

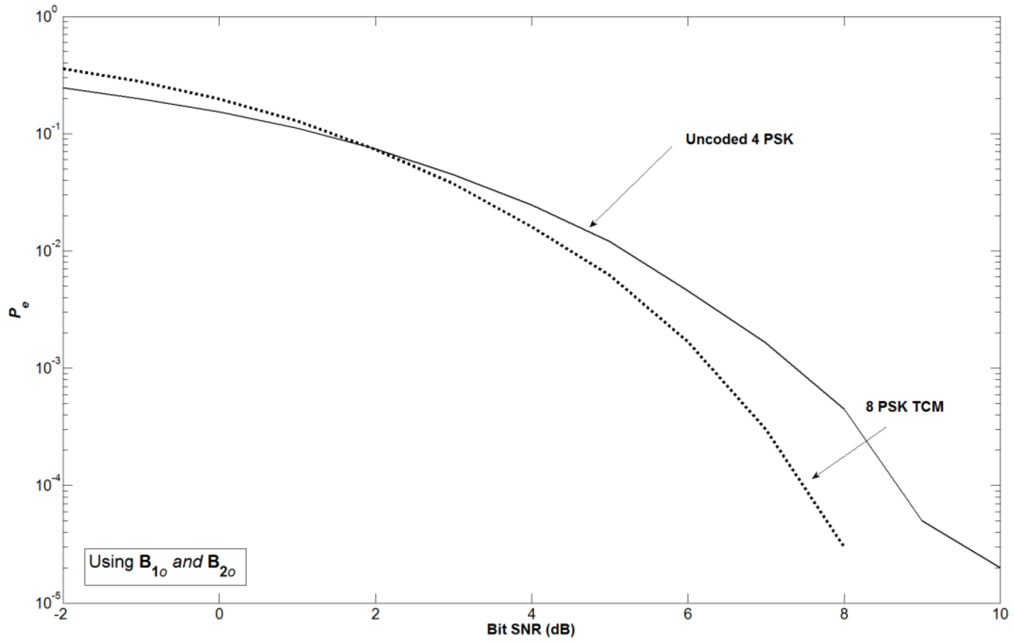

Fig. 1.3 Fig. 1.4 Probability of error curves for uncoded 4 PSK and for 8 PSK TCM using the designation of (1.5).

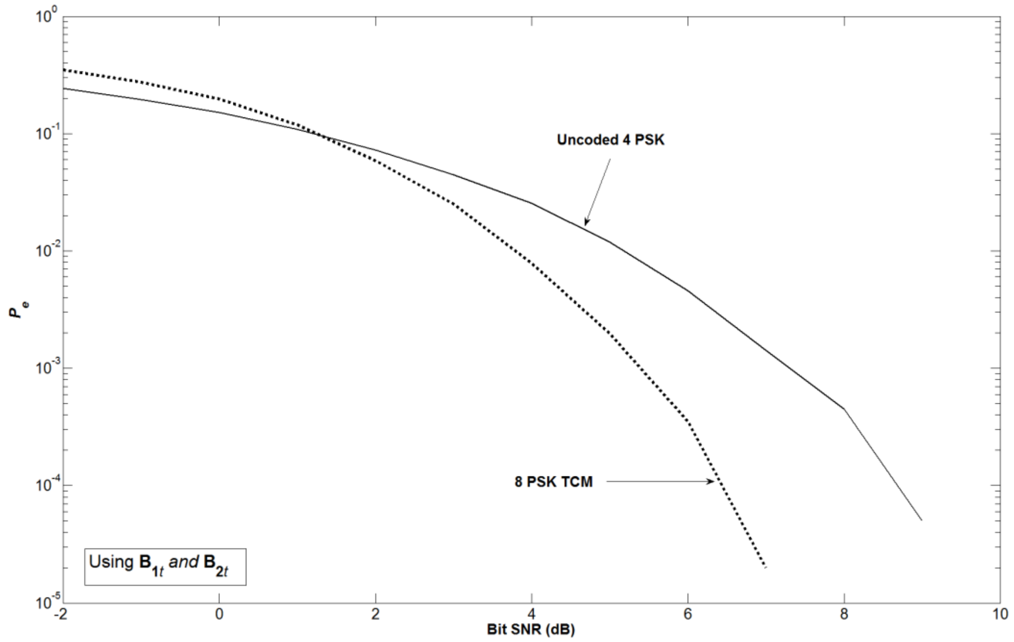

Fig. 1.4 Probability of error curves for uncoded 4 PSK and for 8 PSK TCM using the designation of (1.6).

2. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones justify your answer.

a) In cyclic codes, cyclic property can be tested using Hamming weights of the code words : False, cyclic property can be tested as explained on page 9 of lecture notes entitled, "ECE 587_Notes on Codes" and as stated in the sentence, "A cyclic code is a linear block code with the extra property that for any code word $\mathbf{c}_i$, its cyclic shift is also a code word in the same code.".

b) In convolutional codes, output depends on the five previous bits of the input sequence : This is only true, if the constraint length is, $L \geq 6$.

c) Linearity property can be tested using modulo-2 addition rules : True, as demonstrated in (2.6) on page 5 of lecture notes entitled, "ECE 587_Notes on Codes".

d) Generator matrix is derived from generator polynomials : True, as shown on pages 12 and 13 of lecture notes entitled, "ECE 587_Notes on Codes".

e) In the code $C = \quad 00010,\ 10110,\ 01111, 11011$, minimum weight is,

$w_{\min} = \min\limits_{c_i \neq 0}\left[w(\mathbf{c}_i)\right] = 2$: False, $w_{\min} = \min\limits_{c_i \neq 0}\left[w(\mathbf{c}_i)\right] = w(\mathbf{c}_1) = 1$

f) The code $C = \quad 01010,\ 10110,\ 01111, 11011$ is a linear block code : False, since

$$\begin{array}{rll} \mathbf{c}_1 \oplus \mathbf{c}_2 = 0\,1\,0\,1\,0 & \leftarrow & \text{Componentwise} \\ \underline{1\,0\,1\,1\,0} & \leftarrow & \text{modulo-2 addition} \\ 1\,1\,1\,0\,0 & \neq \mathbf{c}_i & \quad i = 1 \cdots 4 \end{array}$$