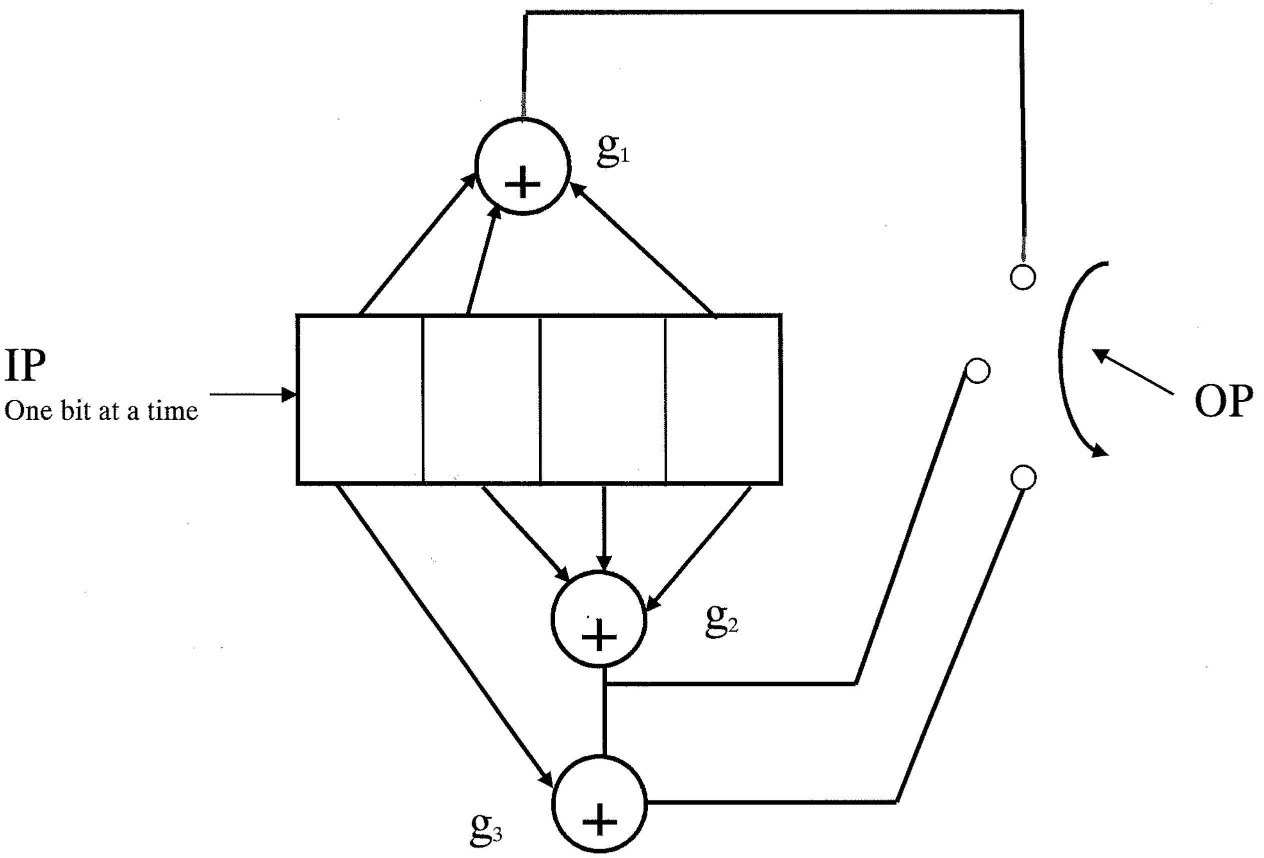
Çankaya University – ECE Department – ECE 587

Student Name :
Student Number :

Duration : 2 hours
Open book exam

Questions

1. (35 Points) Following convolutional encoder is given. IP is one bit at a time. Determine the k, n, code rate (k / n) and constrain length (L). Write for the generator sequences ($\mathbf{g}_1$, $\mathbf{g}_2$, $\mathbf{g}_3$). Draw the state and trellis diagrams and construct the convolutional encoder in MATLAB. If input sequence of 1100101101 is fed to this encoder, find the output of the encoder, by interlacing and from MATLAB. Feed the same OP to the decoder in MATLAB and test to see if you get the given input. Copy the MATLAB code on this exam paper



Solution: From the given values and from the figure, we have $k = 1$, $n = 3$, $k/n = 1/3$

$L = 4$

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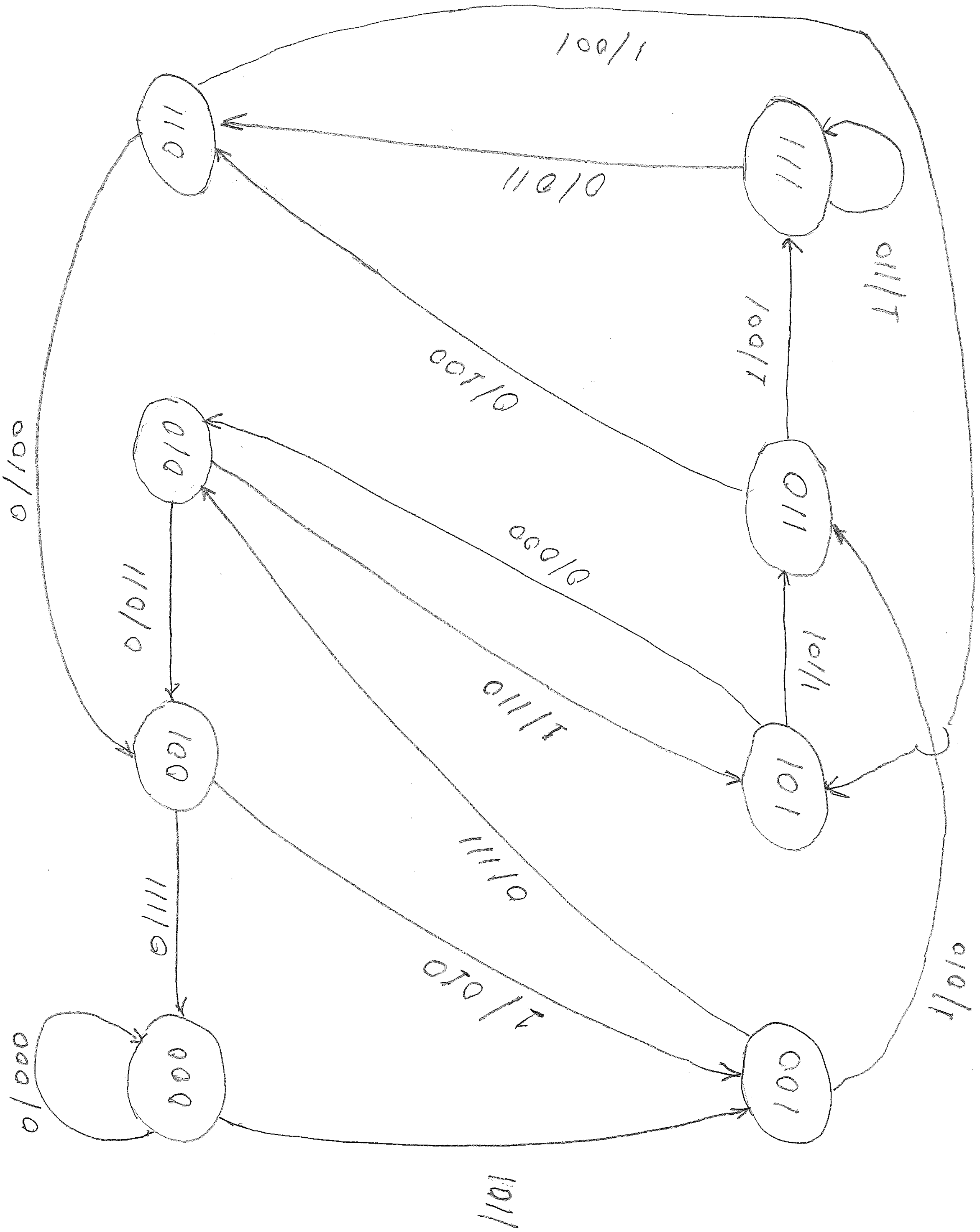
The state diagram is shown below



Now for I/P sequence of $\overset{\nwarrow p^9}{1}10010110\overset{\nearrow p^0}{1}$

$$X(p) = p^9 + p^8 + p^5 + p^3 + p^2 + 1$$

we find OP by interlacing method

$g_1 = [1\ \ 1\ \ 0\ \ 1] = \overset{\text{Decimal}}{13} = \overset{\text{Octal}}{15}$

$g_1(p) = p^3 + p^2 + 1$

$g_2 = [0\ \ 1\ \ 1\ \ 1] = \overset{\text{Decimal}}{7} = \overset{\text{Octal}}{7}$

$g_2(p) = p^2 + p + 1$

$g_3 = [1\ \ 1\ \ 1\ \ 1] = \overset{\text{Decimal}}{15} = \overset{\text{Octal}}{17}$

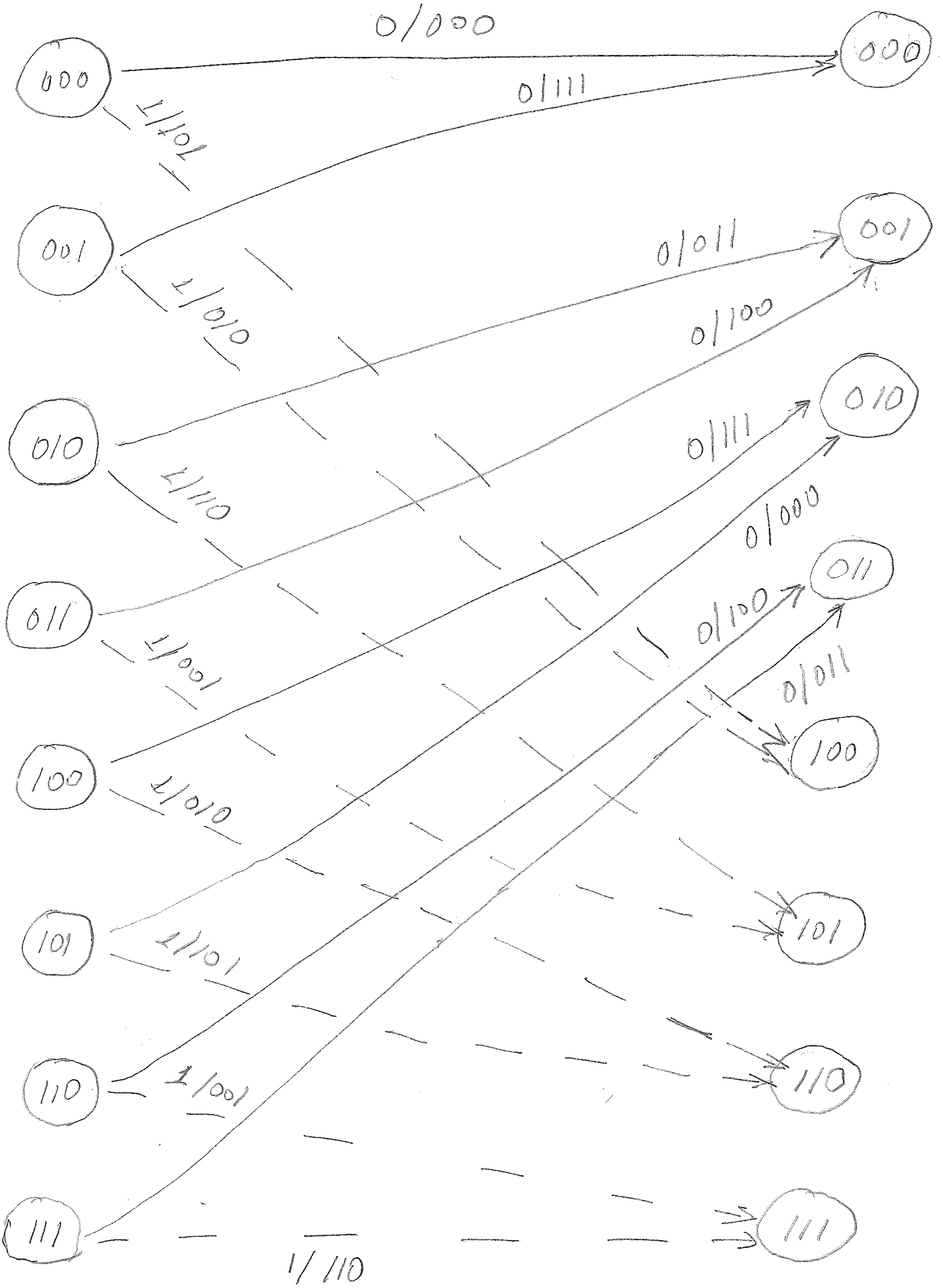
$X(p)\,g_1(p) = p^{12} + p^{10} + p^9 + p^7 + p^6 + p^4 + 1$

$X(p)\,g_2(p) = p^{11} + p^8 + p^7 + p^6 + p + 1$

$X(p)\,g_3(p) = p^{12} + p^7 + p^5 + p^3 + p + 1$

Interlace and put in the order given above to get

Two stages of Trellis Diagram for the given convolutional coder is shown below



Encoded OP =

p^{12}	p^{11}	p^{10}	p^{9}
101	010	100	100

p^{8}	p^{7}	p^{6}	p^{5}
010	111	110	101

p^{4}	p^{3}	p^{2}	p^{1}	p^{0}
100	001	000	011	111

By constructing the encoder and decoder in MATLAB as shown overleaf

where

$$t = \text{polytrellis}(\overset{\overset{L}{\uparrow}}{4}, [\overset{\overset{g_1}{\uparrow}}{15} \;\; \overset{\overset{g_2}{\uparrow}}{7} \;\; \overset{\overset{g_3 \leftarrow \text{in octal form}}{\uparrow}}{17}])$$

by writting Message = [1 1 0 0 1 0 1 1 0]

we get

$$OP = [\overbrace{1\,0\,1}^{p^{12}} \;\; \overbrace{0\,1\,0}^{p^{11}} \;\; \overbrace{1\,0\,0}^{p^{10}} \;\; \overbrace{1\,0\,0}^{p^{9}} \;\; \overbrace{0\,1\,0}^{p^{8}}$$

$$\overbrace{1\,1\,1}^{p^{7}} \;\; \overbrace{1\,1\,0}^{p^{6}} \;\; \overbrace{1\,0\,1}^{p^{5}} \;\; \overbrace{1\,0\,0}^{p^{4}} \;\; \overbrace{0\,0\,1}^{p^{3}}$$

As seen in the (interlaced) output there are three extra codewords (named p^2, p^1, p^0) which are used to flush the registers to all zero state. Such codewords are absent in MATLAB OP. Note that the same MATLAB code can be used to determine the OP of state transition diagram or Trellis Diagram by feeding in the message as

$$\text{Message} = [\underbrace{0\ \ 0\ \ 1}_{\text{To bring the encoder to the state of 100}}\ \ \underbrace{0}_{\text{Inputting 0}}]$$

In this case encoded OP will be

← Previous OPs →

$$\underbrace{0\ 0\ 0}\ \underbrace{0\ 0\ 0}\ \underbrace{1\ 0\ 1}\ \underbrace{1\ 1\ 1}_{\text{Desired OP}}$$

2. (35 Points) Consider a (7,4) linear block code with the generator **G** matrix as shown below. Find all the codewords (C) of this code. Find the parity check matrix of this code **H**. For all the codewords in C, compute the Hamming distance, $d(\mathbf{c}_i, \mathbf{c}_j)$, Hamming weight $w(\mathbf{c}_i)$, minimum distance, d_{min}, minimum weight, w_{min} verifying that $d_{min} = w_{min}$. .

$$\mathbf{G} = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

Solution: G is in systematic form, where $n = 7$, $k = 4$

Assume the message signal is $X = \overset{\longleftarrow\ k=4\ \longrightarrow}{[x_1, x_2, x_3, x_4]}$

To find code words we set $c = XG$, hence

$c = [c_1, c_2, c_3, c_4, c_5, c_6, c_7]$, where

$c_1 = x_1$, $c_2 = x_2$, $c_3 = x_3$, $c_4 = x_4$

$c_5 = x_1 \oplus x_2 \oplus x_4$, $c_6 = x_1 \oplus x_3 \oplus x_4$, $c_7 = x_1 \oplus x_2 \oplus x_3$

$H = [-P^t / I_{n-k}]$ in binary $-P^t = P^t$, $t \rightarrow$ transpose

$$P\text{(the last three columns of } G) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Thus $p^t = \begin{bmatrix} 1101 \\ 1011 \\ 1110 \end{bmatrix}$, this way H will become

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

To find all code words in C, we have to compute $c = x\,G$, where x= de2bi(0:15) in MATLAB notation. This is shoon in the answers of ECE 587 MT Exam 28.11.2008 Q2.

3. (30 Points) Answer the following questions as **True** or **False**. For the **False** ones give the correct answer or the reason. For the **True** ones justify your answer.

a) Coding increases signal to noise ration (SNR) : Not exactly, it is the seperation between signal vectors becoming greater

b) In OFDM, there are four subcarriers : False, there is no such limitation, generally since OFDM is derived from Mary PSK or QAM, number of subcarriers is chosen to be M

c) In OFDM subcarriers are orthogonal to each other along time axis : within a symbol interval of T_s

$$\int_0^{T_s} \cos 2\pi f_p t \cos 2\pi f_m t \, dt \begin{cases} \rightarrow = 0 \text{ if } p \neq m \\ \rightarrow \neq 0 \text{ if } p = m \end{cases}$$

d) OFDM give protection against fading channels : True

OFDM is the protection against frequency selective channel fadings

e) In OFDM, modulation and demodulation are achieved via integration : False

Modulation is via multiplication

Demodulation is via integration

f) OFDM can be used for video type of message signals : False

There is no such restriction, message signal can be of any type.