Çankaya University – ECE Department – ECE 587

Student Name : Open source exam
Student Number : Duration : 2 hours

Questions

1. (40 Points) A 128 Mbits/sec signal is given. This signal will be modulated to 4 PSK and 8 PSK and will then be transmitted using OFDM with the number of subcarriers equal to the Mary value of each modulation. Find the orthogonally arranged OFDM subcarriers frequencies for each case. Scaling (dividing) the subcarriers by 10^6 and working in discrete Fourier transforms, show the successful demodulation of the symbol on the third subcarrier for each case. You may benefit from the use of the Matlab file, Calculations_Q1_FE_Jan2016_Sample.m, available on the course webpage.

Solution : For a 128 Mbits/sec signal, $T_b = 7.8125 \text{ nsec}$, upon converting this signal into 4 PSK, we have, $T_{s4PSK} = T_b \times \log_2 M = T_b \times \log_2 4 = 15.625 \text{ nsec}$. Then for (parallel) OFDM symbol, we have $T_{4PSK} = MT_{s4PSK} = 4 \times T_{s4PSK} = 62.5 \text{ nsec}$. Doing the same 8 PSK, we have, $T_{s8PSK} = T_b \times \log_2 M = T_b \times \log_2 8 = 23.438 \text{ nsec}$, $T_{8PSK} = MT_{s8PSK} = 8 \times T_{s8PSK} = 187.5 \text{ nsec}$. This way, the OFDM subcarriers are evaluated as .

$$
\begin{aligned}
&f_{1\_4PSK} = \frac{1}{T_{4PSK}} = 16 \text{ MHz} \,,\; f_{2\_4PSK} = \frac{2}{T_{4PSK}} = 32 \text{ MHz} \\
&f_{3\_4PSK} = \frac{3}{T_{4PSK}} = 48 \text{ MHz} \,,\; f_{4\_4PSK} = \frac{4}{T_{4PSK}} = 64 \text{ MHz} \\
&f_{1\_8PSK} = \frac{3}{T_{8PSK}} = 16 \text{ MHz} \,,\; f_{2\_8PSK} = \frac{6}{T_{8PSK}} = 32 \text{ MHz} \\
&f_{3\_8PSK} = \frac{9}{T_{8PSK}} = 48 \text{ MHz} \,,\; f_{4\_8PSK} = \frac{12}{T_{8PSK}} = 64 \text{ MHz} \\
&f_{5\_8PSK} = \frac{15}{T_{8PSK}} = 80 \text{ MHz} \,,\; f_{6\_8PSK} = \frac{18}{T_{8PSK}} = 96 \text{ MHz} \\
&f_{7\_8PSK} = \frac{21}{T_{8PSK}} = 112 \text{ MHz} \,,\; f_{8\_8PSK} = \frac{24}{T_{8PSK}} = 128 \text{ MHz} \\
&c_k(t) = \cos(2\pi f_k t) \quad , \quad 1 \le k \le K = M = 4 \text{ for 4PSK} \\
&\qquad\qquad\qquad\qquad\;\; 1 \le k \le K = M = 8 \text{ for 8PSK}
\end{aligned}
\tag{1.1}
$$

Note that the subcarriers of 8 PSK in (1.1) are selected to be integers. The symbols in the constellations of 4 PSK and 8 PSK are given in (1.2)

$$\mathbf{s}_{1\_4PSK} = 1,\ \mathbf{s}_{2\_4PSK} = j,\ \mathbf{s}_{3\_4PSK} = -1,\ \mathbf{s}_{4\_4PSK} = -j$$

$$\mathbf{s}_{1\_8PSK} = 1,\ \mathbf{s}_{2\_8PSK} = \frac{1+j}{\sqrt{2}},\ \mathbf{s}_{3\_8PSK} = j,\ \mathbf{s}_{4\_4PSK} = \frac{-1+j}{\sqrt{2}}$$

$$\mathbf{s}_{5\_8PSK} = -1,\ \mathbf{s}_{6\_8PSK} = \frac{-1-j}{\sqrt{2}},\ \mathbf{s}_{7\_8PSK} = -j,\ \mathbf{s}_{8\_4PSK} = \frac{1-j}{\sqrt{2}} \quad (1.2)$$

Upon scaling the subcarriers in (1.1) by 10^6, we have

$$f_{1d\_4PSK} = \frac{10^{-6}}{T_{4PSK}} = 16 \text{ Hz},\ f_{2d\_4PSK} = \frac{2\times10^{-6}}{T_{4PSK}} = 32 \text{ Hz}$$

$$f_{3d\_4PSK} = \frac{3\times10^{-6}}{T_{4PSK}} = 48 \text{ Hz},\ f_{4d\_4PSK} = \frac{4\times10^{-6}}{T_{4PSK}} = 64 \text{ Hz}$$

$$f_{1d\_8PSK} = \frac{3\times10^{-6}}{T_{8PSK}} = 16 \text{ Hz},\ f_{2\_8PSK} = \frac{6\times10^{-6}}{T_{8PSK}} = 32 \text{ Hz}$$

$$f_{3d\_8PSK} = \frac{9\times10^{-6}}{T_{8PSK}} = 48 \text{ Hz},\ f_{4d\_8PSK} = \frac{12\times10^{-6}}{T_{8PSK}} = 64 \text{ Hz}$$

$$f_{5d\_8PSK} = \frac{15\times10^{-6}}{T_{8PSK}} = 80 \text{ Hz},\ f_{6d\_8PSK} = \frac{18\times10^{-6}}{T_{8PSK}} = 96 \text{ Hz}$$

$$f_{7d\_8PSK} = \frac{21\times10^{-6}}{T_{8PSK}} = 112 \text{ Hz},\ f_{8d\_8PSK} = \frac{24\times10^{-6}}{T_{8PSK}} = 128 \text{ Hz}$$

$$c_k(t) = \exp(j2\pi f_k t) \quad , \quad 1 \le k \le K = M = 4 \text{ for 4PSK}$$

$$1 \le k \le K = M = 8 \text{ for 8PSK} \quad (1.3)$$

Taking one OFDM symbol interval, we have on the transmitter side, after modulation

$$y_{4PSK}(n) = \frac{1}{N}\sum_{k=1}^{K} {}_k\mathbf{s}_{m\_4PSK} \exp\left(j2\pi f_{kd\_4PSK} n/N\right) = \frac{1}{N}\sum_{k=1}^{4} {}_k\mathbf{s}_{m\_4PSK} \exp\left(j2\pi f_{kd\_4PSK} n/N\right)$$

$$N = 64,\ m = 1\cdots4,\ n = 0\cdots63$$

$$y_{8PSK}(n) = \frac{1}{N}\sum_{k=1}^{K} {}_k\mathbf{s}_{m\_8PSK} \exp\left(j2\pi f_{kd\_8PSK} n/N\right) = \frac{1}{N}\sum_{k=1}^{4} {}_k\mathbf{s}_{m\_8PSK} \exp\left(j2\pi f_{kd\_8PSK} n/N\right)$$

$$N = 128,\ m = 1\cdots8,\ n = 0\cdots127 \quad (1.4)$$

We choose the following arbitrary assignment of PSK symbols for 4 PSK and 8 PSK respectively

$${}_1\mathbf{s}_{m\_4PSK} = \mathbf{s}_{1\_4PSK},\ {}_2\mathbf{s}_{m\_4PSK} = \mathbf{s}_{4\_4PSK},\ {}_3\mathbf{s}_{m\_4PSK} = \mathbf{s}_{2\_4PSK},\ {}_4\mathbf{s}_{m\_4PSK} = \mathbf{s}_{1\_4PSK}$$

$${}_1\mathbf{s}_{m\_8PSK} = \mathbf{s}_{1\_8PSK},\ {}_2\mathbf{s}_{m\_8PSK} = \mathbf{s}_{4\_4PSK},\ {}_3\mathbf{s}_{m\_8PSK} = \mathbf{s}_{2\_8PSK},\ {}_4\mathbf{s}_{m\_8PSK} = \mathbf{s}_{1\_8PSK}$$

$${}_5\mathbf{s}_{m\_8PSK} = \mathbf{s}_{1\_8PSK},\ {}_6\mathbf{s}_{m\_8PSK} = \mathbf{s}_{7\_4PSK},\ {}_7\mathbf{s}_{m\_8PSK} = \mathbf{s}_{8\_8PSK},\ {}_8\mathbf{s}_{m\_8PSK} = \mathbf{s}_{5\_8PSK} \quad (1.5)$$

Then for demodulation of the first symbols on the receiver side

$$d_{3\_4PSK} = \sum_{n=0}^{N-1} y_{4PSK}(n) \exp\left(-j2\pi f_{3d\_4PSK} n/N\right) = 0 + 0 + \mathbf{s}_{2\_4PSK} + 0 = j$$

$$d_{3\_8PSK} = \sum_{n=0}^{N-1} y_{8PSK}(n) \exp\left(-j2\pi f_{3d\_8PSK} n/N\right) = 0 + 0 + \mathbf{s}_{2\_8PSK} + 0 + 0 + 0 + 0 + 0 = \frac{1+j}{\sqrt{2}} \quad (1.6)$$

The calculations are performed in the Matlab file, Calculations_Q1_FE_Jan2016.m.

2. (25 Points) The message signal of a DS spread spectrum system uses a PN sequence of $T_c = 1\ \mu\text{sec}$ with a processing gain of $L_c = 1000$. Estimate the approximate bandwidths of this DS signal before and after spreading. Find the power of an interfering signal that that will give a signal to interference ratio, $\text{SIR} = -10\ \text{dB}$ after the demodulation (despreading) operation on the receiver side. Assume that DS spread spectrum signal at receiver has a symbol amplitude of 1 mV. Describe what type of signals can be regarded as interfering signals to this DS spread spectrum system and write the mathematical expression of these interfering signals.

Solution : From $T_b / T_c = L_c$, we find $T_b = T_c L_c = 10^{-6} \times 1000 = 10^{-3}\ \text{sec} = 1\ \text{msec}$. Hence we find, the bandwidths before and after spreading as

$$\text{Bandwidth before spreading, } B_V = 1/T_b - \left(-1/T_b\right) = 2/T_b = 2\ \text{kHz}$$
$$\text{Bandwidth after spreading, } B_S = 1/T_c + 1/T_b - \left(-1/T_c - 1/T_b\right) = 2.002\ \text{MHz} \qquad (2.1)$$

For SIR calculations, we set SIR on receiver side as follows

After demodulation

Signal power : $P_s = \quad = \quad =$

$$\text{SIR} = \frac{P}{I_i / L_c} = \quad \rightarrow \quad = \qquad (2.2)$$

Two examples of interfering signals are given below

$$I_{i1} = 0.1,\ I_{i2} = 0.1\sqrt{2}\cos\left(2\pi f_i t\right),\ f_i \le 1.999\ \text{MHz} \qquad (2.3)$$

3. (35 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones justify your answer.

a) Coding becomes more powerful, if there is dependence on the past : True, for instance convolutional coding that has dependence on the past (as well as the present), is more powerful, thus has better performance than cyclic and linear block codes.

b) The performance of cyclic coding is better than linear block codes : Such a comparison is not shown in the lecture notes. In order to make such an assessment, we need to run simulations at the same code rate.

c) Cyclic autocorrelation graphics of all PN sequences are the same : False, cyclic autocorrelation graphics of maximum length PN sequences are the same named as $\mathbf{e}_1$ and $\mathbf{e}_2$ in section 3 of lecture notes entitled, "Spread spectrum systems_2013_HTE".

d) Desirable PN sequences should have orthogonal cross correlation properties : True, according to page 16 of the lecture notes entitled, "Spread spectrum systems_2013_HTE".

e) Correlation operation is necessary for the successful demodulation of a DS spread spectrum signal : True, as illustrated in Figs. 2.1 and 2.2 of the lecture notes entitled, "Spread spectrum systems_2013_HTE".

f) In spread spectrum systems, spreading is achieved by PN sequences who have lower bandwidths than the message signal : The first part is correct, but the bandwidths of PN sequences are much larger (by an amount of L_c) than the message signal.