Çankaya University – ECE Department – ECE 587

Student Name :
Student Number :

Open source exam
Duration : 2 hours

Questions

1. (35 Points) A 64 Mbits/sec signal is given. This signal is modulated to 16 QAM and transmitted using OFDM with number of subcarriers equal to M ary, i.e. $M = 16 = K \leftarrow$ number of OFDM subcarriers Find the sixteen orthogonally arranged OFDM subcarriers frequencies. Draw the approximate the corresponding OFDM time waveform and the frequency spectrum, by taking the initial four 16 QAM symbols as $\mathbf{s}_1,\ \mathbf{s}_6,\ \mathbf{s}_{16},\ \mathbf{s}_9$. Note that you can use Matlab model block to get 16 QAM signals as

$$\begin{aligned}
&\mathbf{s}_1 = -3+3j,\ \mathbf{s}_2 = -3+j,\ \mathbf{s}_3 = -3-3j,\ \mathbf{s}_4 = -3-j \\
&\mathbf{s}_5 = -1+3j,\ \mathbf{s}_6 = -1+j,\ \mathbf{s}_7 = -1-3j,\ \mathbf{s}_8 = -1-j \\
&\mathbf{s}_9 = 3+3j,\ \mathbf{s}_{10} = 3+j,\ \mathbf{s}_{11} = 3-3j,\ \mathbf{s}_{12} = 3-j \\
&\mathbf{s}_{13} = 1+3j,\ \mathbf{s}_{14} = 1+j,\ \mathbf{s}_{15} = 1-3j,\ \mathbf{s}_{16} = 1-j
\end{aligned} \tag{1.1}$$

Write for the transmitted OFDM signal $y(t)$ and show the successful demodulation of the 16 QAM signal on the third subcarrier.

Solution : For a 64 Mbits/sec signal, $T_b = 1/64\ \mu\text{sec}$, upon converting this signal into 16 QAM, we have, $T_s = T_b \times \log_2 M = T_b \times \log_2 16 = 1/16\ \mu\text{sec}$. After applying OFDM to this 16 QAM signal, $T = MT_s = KT_s = 1\ \mu\text{sec}$. This way, the OFDM subcarriers are evaluated as .

$$\begin{aligned}
&f_1 = \frac{1}{T} = 1 \text{ MHz},\ f_2 = \frac{2}{T} = 2 \text{ MHz},\ f_3 = \frac{3}{T} = 3 \text{ MHz},\ f_4 = \frac{4}{T} = 4 \text{ MHz} \\
&f_5 = \frac{5}{T} = 5 \text{ MHz},\ f_6 = \frac{6}{T} = 6 \text{ MHz},\ f_7 = \frac{7}{T} = 7 \text{ MHz},\ f_8 = \frac{8}{T} = 8 \text{ MHz} \\
&f_9 = \frac{9}{T} = 9 \text{ MHz},\ f_{10} = \frac{10}{T} = 10 \text{ MHz},\ f_{11} = \frac{11}{T} = 11 \text{ MHz},\ f_{12} = \frac{12}{T} = 12 \text{ MHz} \\
&f_{13} = \frac{13}{T} = 13 \text{ MHz},\ f_{14} = \frac{14}{T} = 14 \text{ MHz},\ f_{15} = \frac{15}{T} = 15 \text{ MHz},\ f_{16} = \frac{16}{T} = 16 \text{ MHz} \\
&c_k(t) = \cos(2\pi f_k t) \quad , \quad 1 \le k \le K = M
\end{aligned} \tag{1.2}$$

The constellation of (1.1) is given in Fig. 1.1. By taking the first four symbols as $\mathbf{s}_1,\ \mathbf{s}_6,\ \mathbf{s}_{16},\ \mathbf{s}_9$, we write for $y(t)$ as follows

$$\begin{aligned}
&y(t) = \sum_{k=1}^{K} y_k(t) = \sum_{k=1}^{K} {}_k\mathbf{s}_m c_k(t) = \sum_{k=1}^{K} {}_k\mathbf{s}_m \cos(2\pi f_k t) \quad , \quad 1 \le m \le M \quad , \quad 1 \le k \le K = M \\
&{}_1\mathbf{s}_m = \mathbf{s}_1,\ {}_2\mathbf{s}_m = \mathbf{s}_6,\ {}_3\mathbf{s}_m = \mathbf{s}_{16},\ {}_4\mathbf{s}_m = \mathbf{s}_9
\end{aligned} \tag{1.2}$$

The time waveform for the four initial symbols is given in Fig. 1.2.

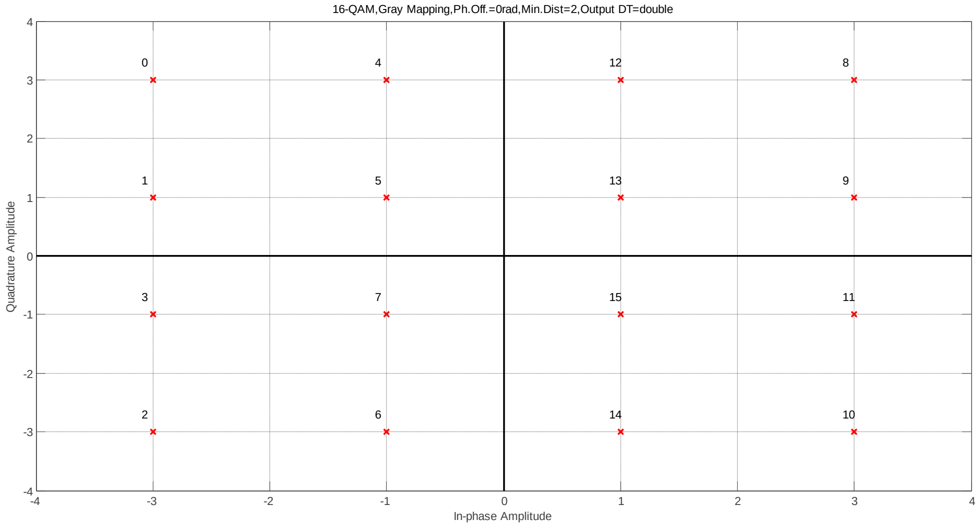

Fig. 1.1 16 QAM constellation taken from Matlab model block, note that the symbols in (1.1) have the indexing of one less.

$y(t)$ is shown for the real and imaginary parts separately in Figs. 1.2a and Fig. 1.2b.

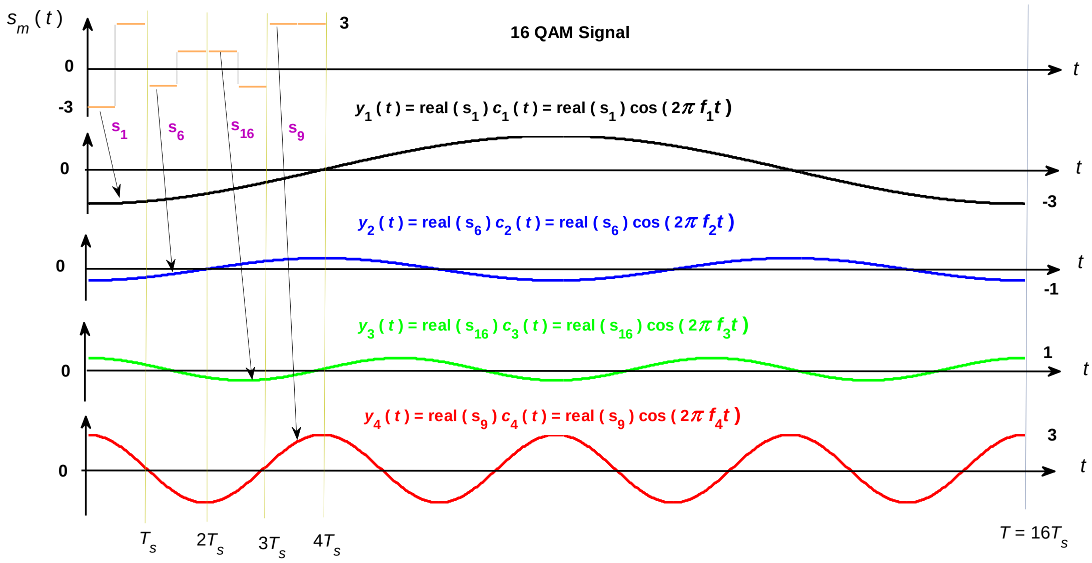

Fig. 1.2a $y(t)$ for real part of the 16 QAM signal.

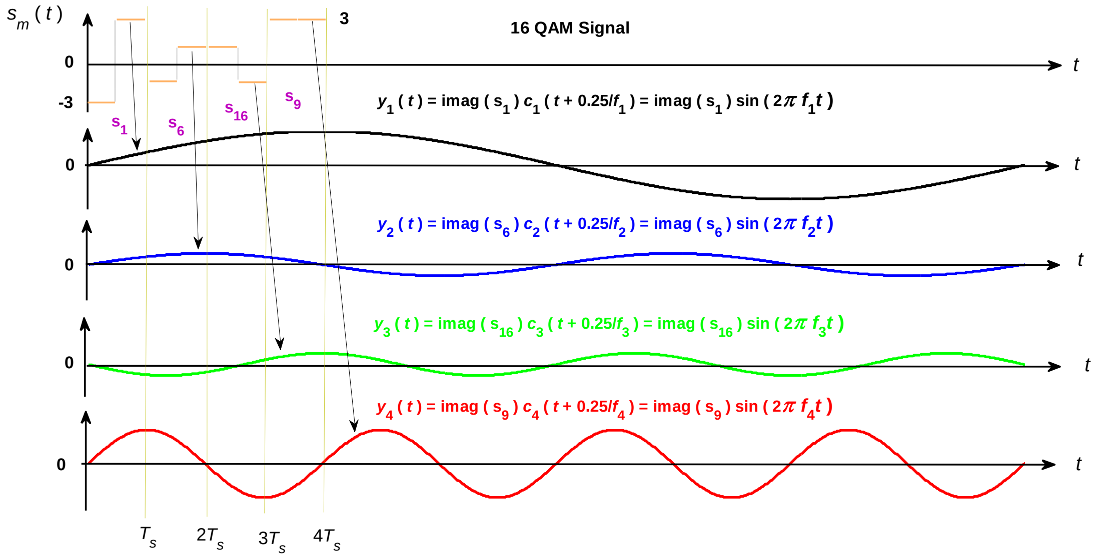

Fig. 1.2b $y(t)$ for imaginary part of the 16 QAM signal.

$Y(f)$ for the first four symbols is given in Fig. 1.3.

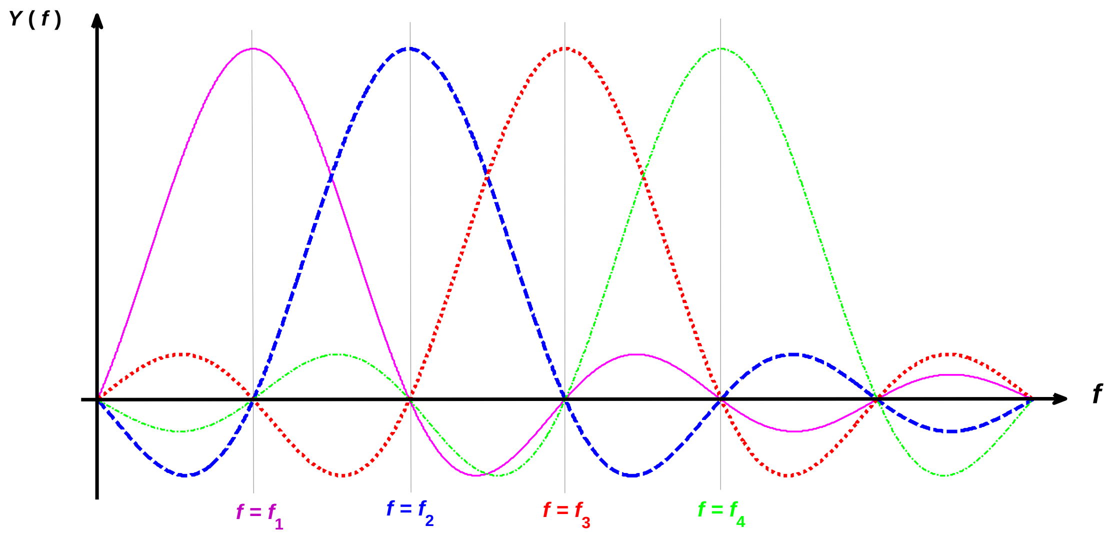

Fig. 1.3 $Y(f)$ for the first four symbols of the 16 QAM signal.

For demodulation on the third subcarrier, we take the transmitted OFDM signal as follows

$$y(n) = \frac{1}{N}\sum_{k=1}^{K} {}_k\mathbf{s}_m \exp\left(j2\pi f_k n / N\right) \;,\quad N = 16 \;,\quad f_{1 \cdots 16} = 1 \cdots 16 \;,\quad n = 0 \cdots 15 \qquad (1.3)$$

During demodulation, we will have

$$
\begin{aligned}
d_3 &= \sum_{n=0}^{N-1} y(n) \exp\left(-j2\pi f_3 n / N\right) = \\
&= \frac{1}{N}\Bigg[\overbrace{\sum_{n=0}^{N-1} {}_1\mathbf{s}_1 \exp\left(j2\pi f_1 n / N\right)\exp\left(-j2\pi f_k n / N\right)}^{0} + \overbrace{\sum_{n=0}^{N-1} {}_2\mathbf{s}_6 \exp\left(j2\pi f_2 n / N\right)\exp\left(-j2\pi f_k n / N\right)}^{0} \\
&\quad + \overbrace{\sum_{n=0}^{N-1} {}_3\mathbf{s}_{16} \exp\left(j2\pi f_k n / N\right)\exp\left(-j2\pi f_k n / N\right)}^{3\,{}_3\mathbf{s}_{16}} \cdots \overbrace{\sum_{n=0}^{N-1} {}_k\mathbf{s}_m \exp\left(j2\pi f_K n / N\right)\exp\left(-j2\pi f_k n / N\right)}^{0} \Bigg] \\
&= 0 + 0 \cdots + \mathbf{s}_{16} \cdots 0 = 1 - j \qquad (1.4)
\end{aligned}
$$

Note that in Matlab implementation (1.4) is represented as matrix multiplication. Here care must be taken since, Matlab incorporates the conjugate in the transpose operation. The calculations are performed in the Matlab file, Calculations_Q1_FE_2013.m.

2. (35 Points) The message signal of a DS spread spectrum system works at $T_b = 0.2$ msec and uses a PN sequence of $T_c = 0.5\ \mu\text{sec}$. Plot the following graphs,

   a) The spectrums of message signal prior to and after spreading,
   b) Time waveforms of message signal prior to and after spreading,

If an interference signal $I_i = 5\cos(20000\pi t) + 50\cos(4\times10^7\pi t)$ is mixed with received signal, find the signal power to interference power ratio (SIR) prior to demodulation (despreading) and after demodulation, assuming that the message signal and the PN spreading sequence at transmitter have unity amplitude and the communication channel has a unity frequency response over the band of related frequencies.

Solution : By taking $1/T_b = 5$ kHz, $1/T_c = 2$ MHz, the spectrums and the time waveforms before and after spreading are shown in Figs 2.1 and 2.2

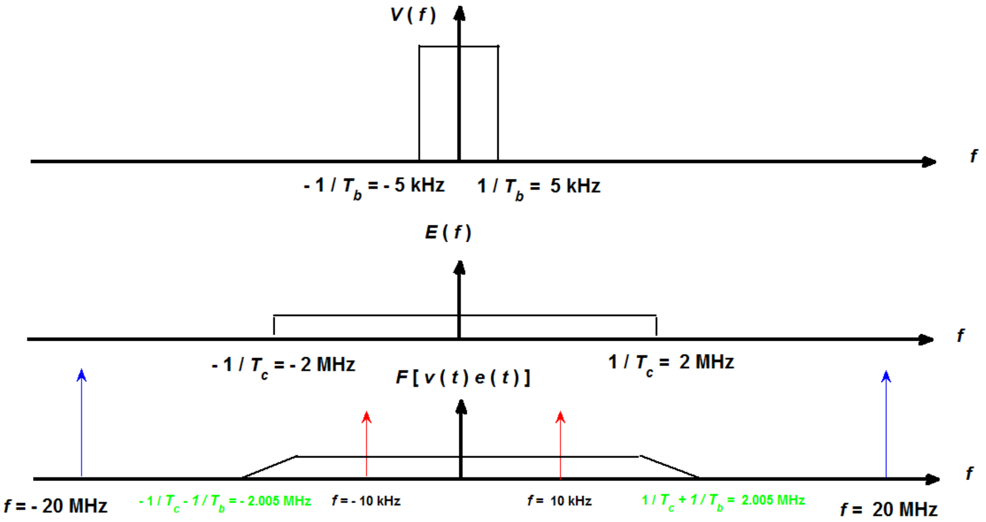

Fig. 2.1 The spectrums of the message signal before and after spreading, where on the last row, the interference signal is also included in the spectral range.

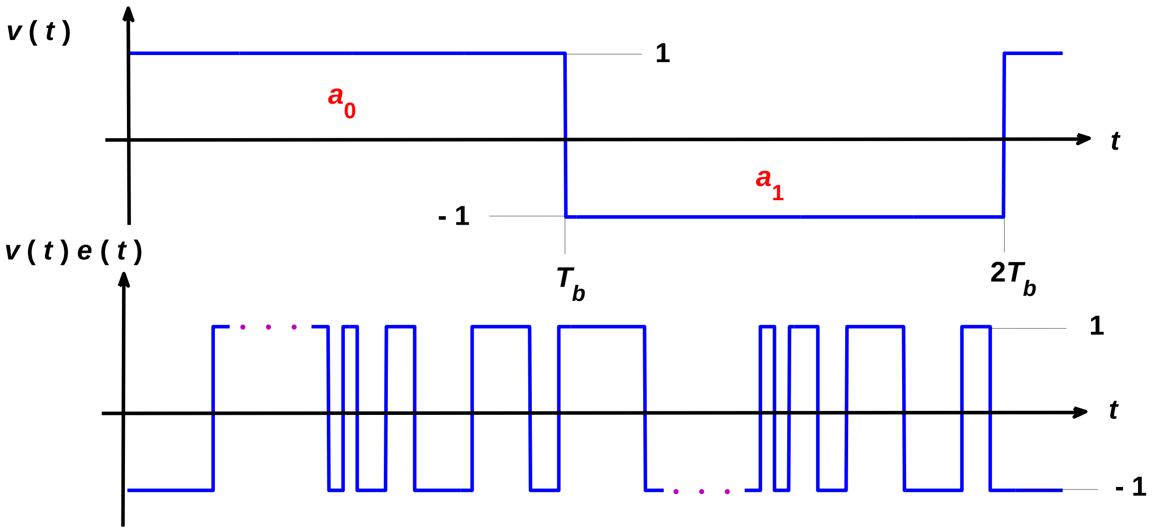

Fig. 2.2 Time waveforms of the message signal before and after spreading.

For SIR calculations, we exclude the second sinusoidal in the interference signal, since it is outside the bandwidth of the spread signal. Hence

Prior to demodulation

Signal power : $P_s = a_0^2 = 1 \text{ W}$, Interference power : $P_i = \left(5/\sqrt{2}\right)^2 = 12.5 \text{ W}$

$$\text{SIR} = \frac{P_s}{P_i} = \frac{1}{12.5} = 0.08 \quad \text{or} \quad -10.97 \text{ dB} \qquad (2.1)$$

After demodulation

Signal power : $P_s = a_0^2 = 1 \text{ W}$, Interference power : $P_i = 12.5/L_c = 12.5T_c/T_b = 0.0313 \text{ W}$

$$\text{SIR} = \frac{P_s}{P_i} = \frac{1}{0.0313} = 32 \quad \text{or} \quad \sim 15 \text{ dB} \qquad (2.2)$$

3. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones justify your answer.

a) SS DS system increases the number of bits (binary waveforms) in a symbol : False, SS DS system divides a symbol duration into L_c number of slices with a duration of T_c, where the sequence is itself the PN code. But if T_c is regarded as bit duration, we can say that the number of bits (binary waveforms) in a symbol has increased.

b) SS DS system offers better performance than ASK or PSK : Such a comparison is not shown in the lecture notes. All we can say is that ASK or PSK are examples of narrow band systems, whereas SS DS is an example of spread spectrum systems.

c) Convolutional codes and linear codes both use the same generator matrix : False, convolution codes are described by the convolution encoder structure and its current output depends on the present as well as the past, but in linear codes, the output is determined only by the present input bits.

d) In spreading a message signal, we use another signal which has a bandwidth narrower than the message signal : False, in spreading a message signal (considered to be narrow band), we multiply it by a spreading waveform (PN sequence) which has a much wider bandwidth.

e) A PN sequence is used to despread the received DS signal : True, a PN sequence, phase locked to the one used in the transmitter is also used in the receiver to despread the received DS signal.