Çankaya University – ECE Department – ECE 587

Student Name : Open source exam
Student Number : Duration : 2 hours

Questions

1. (35 Points) An 8 Mbits/sec signal is given. This signal is modulated into 8 PSK and 16 QAM and transmitted using OFDM with number of subcarriers equal to M ary (i.e. $M = 8$ for 8 PSK, $M = 16$ for 16 QAM) level of the modulating scheme. Find the orthogonally arranged OFDM subcarriers frequencies of both cases. Draw the approximate frequency spectrum of the modulated subcarriers for each case, comment on which one is spectrally efficient.

   Upon dividing the orthogonally arranged frequency of the third subcarrier by 2, show by using DFT and IDTF that the orthogonality condition is violated in both cases and the demodulation of the symbol placed on that particular subcarrier is no longer successful.

Keep in mind that for an 8 Mbits/sec signal, $T_b = 0.125\ \mu\text{sec}$, $T_s = \log_2(M)T_b$, $T = MT_s$

Solution : An 8 Mbits/sec signal when 8 PSK modulated, $T_s = T_b \times \log_2 M = 0.375\ \mu\text{sec}$, furthermore, $T = MT_s = 3\ \mu\text{sec} = T_{8\text{PSK}}$.

The same calculations for 16 QAM is , $T_s = T_b \times \log_2 M = 0.5\ \mu\text{sec}$, $T = MT_s = 8\ \mu\text{sec} = T_{16\text{QAM}}$.

The number of subcarriers 8 PSK will be eight, whereas for 16 QAM, it will be sixteen. The frequencies of these two cases are computed as follows

For the case of 8 PSK

$$f_1 = \frac{1}{T_{8\text{PSK}}} = \frac{1}{3}\text{ MHz},\ f_2 = \frac{2}{T_{8\text{PSK}}} = \frac{2}{3}\text{ MHz},\ f_3 = \frac{3}{T_{8\text{PSK}}} = 1\text{ MHz},\ f_4 = \frac{4}{T_{8\text{PSK}}} = \frac{4}{3}\text{ MHz}$$
$$f_5 = \frac{5}{T_{8\text{PSK}}} = \frac{5}{3}\text{ MHz},\ f_6 = \frac{6}{T_{8\text{PSK}}} = 2\text{ MHz},\ f_7 = \frac{7}{T_{8\text{PSK}}} = \frac{7}{3}\text{ MHz},\ f_8 = \frac{8}{T_{8\text{PSK}}} = \frac{8}{3}\text{ MHz} \quad (1.1)$$

For the case of 16 QAM

$$f_1 = \frac{1}{T_{16\text{QAM}}} = \frac{1}{8}\text{ MHz},\ f_2 = \frac{2}{T_{16\text{QAM}}} = \frac{1}{4}\text{ MHz},\ f_3 = \frac{3}{T_{16\text{QAM}}} = \frac{3}{8}\text{ MHz},\ f_4 = \frac{4}{T_{16\text{QAM}}} = \frac{1}{2}\text{ MHz}$$
$$f_5 = \frac{5}{T_{16\text{QAM}}} = \frac{5}{8}\text{ MHz},\ f_6 = \frac{6}{T_{16\text{QAM}}} = \frac{3}{4}\text{ MHz},\ f_7 = \frac{7}{T_{16\text{QAM}}} = \frac{7}{8}\text{ MHz},\ f_8 = \frac{8}{T_{16\text{QAM}}} = 1\text{ MHz}$$
$$f_9 = \frac{9}{T_{16\text{QAM}}} = \frac{9}{8}\text{ MHz},\ f_{10} = \frac{10}{T_{16\text{QAM}}} = \frac{5}{4}\text{ MHz},\ f_{11} = \frac{11}{T_{16\text{QAM}}} = \frac{11}{8}\text{ MHz},\ f_{12} = \frac{12}{T_{16\text{QAM}}} = \frac{3}{2}\text{ MHz}$$
$$f_{13} = \frac{13}{T_{16\text{QAM}}} = \frac{13}{8}\text{ MHz},\ f_{14} = \frac{14}{T_{16\text{QAM}}} = \frac{7}{4}\text{ MHz},\ f_{15} = \frac{15}{T_{16\text{QAM}}} = \frac{15}{8}\text{ MHz},\ f_{16} = \frac{16}{T_{16\text{QAM}}} = 2\text{ MHz} \quad (1.2)$$

The frequency spectrum of 8 PSK subcarriers will be as shown in Fig. 1.1

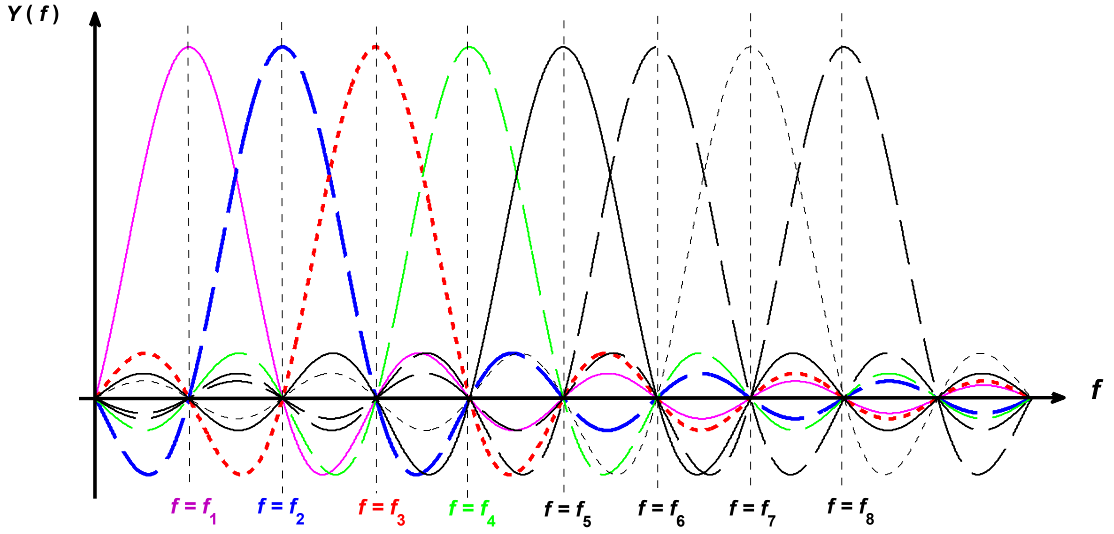

Fig. 1.1. Frequency spectrum of orthogonally arranged 8 PSK subcarriers.

Taking the bandwidth to be the difference between the highest and lowest subcarrier frequencies, we find

For the case of 8 PSK

$$\mathrm{BW}_{8\mathrm{PSK}} = f_8 - f_1 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3} \text{ MHz}$$

For the case of 16 QAM

$$\mathrm{BW}_{16\mathrm{QAM}} = f_{16} - f_1 = 2 - \frac{1}{8} = \frac{15}{8} \text{ MHz} \qquad (1.3)$$

Hence 16 QAM is more efficient spectrally.

From (4.6) of Notes on OFDM_2013, the transmitted OFDM signal becomes,

$$y(n) = \frac{1}{N}\sum_{k=1}^{K} {}_k\mathbf{s}_m \exp\left(j2\pi f_k n / N\right) \qquad (1.4)$$

With the assignments of the subcarrier frequencies being as shown in (1.1) and (1.2) then using (4.4) of Notes on OFDM_2013, the output on the k th arm of the demodulator will be

$$d_k = \sum_{n=0}^{N-1} y(n)\exp\left(-j2\pi f_k n/N\right) =$$

$$= \frac{1}{N}\left[\overbrace{\sum_{n=0}^{N-1} {}_1\mathbf{s}_m \exp\left(j2\pi f_1 n/N\right)\exp\left(-j2\pi f_k n/N\right)}^{0} + \overbrace{\sum_{n=0}^{N-1} {}_2\mathbf{s}_m \exp\left(j2\pi f_2 n/N\right)\exp\left(-j2\pi f_k n/N\right)}^{0} \right.$$

$$\left. \cdots \overbrace{\sum_{n=0}^{N-1} {}_k\mathbf{s}_m \exp\left(j2\pi f_k n/N\right)\exp\left(-j2\pi f_k n/N\right)}^{N\,{}_k\mathbf{s}_m} \cdots \overbrace{\sum_{n=0}^{N-1} {}_k\mathbf{s}_m \exp\left(j2\pi f_K n/N\right)\exp\left(-j2\pi f_k n/N\right)}^{0} \right]$$

$$= 0 + 0 \cdots + {}_k\mathbf{s}_m \cdots 0 \tag{1.5}$$

Note that the operation in (1.5) is successful only if the subcarrier frequencies are integer values, whilst adhering to the rule of orthogonality. The simplest way to achieve this is to multiply to the frequency values in (1.1) by 3 and those in (1.2) by 8. Note that these comments are only applicable to discreet case. In the analytic case, there is no such restriction. In summary, the intended symbol of 8 PSK or 16 QAM can be successfully demodulated, provided that the subcarrier frequencies are assigned as in (1.1) and (1.2) with such restrictions.

Below, we show the case of successfully demodulation for the third subcarrier of 8 PSK, when subcarrier frequencies are as those in (1.1) but multiplied by 3

$$d_3 = \frac{1}{N}\left[\overbrace{\sum_{n=0}^{N-1} {}_1\mathbf{s}_m \exp\left(j2\pi \times 10^6 \times n/N\right)\exp\left(-j2\pi \times 3\times 10^6 \times n/N\right)}^{0} \right.$$

$$+ \overbrace{\sum_{n=0}^{N-1} {}_2\mathbf{s}_m \exp\left(j2\pi \times 2\times 10^6 \times n/N\right)\exp\left(-j2\pi \times 3\times 10^6 \times n/N\right)}^{0}$$

$$+ \overbrace{\sum_{n=0}^{N-1} {}_3\mathbf{s}_m \exp\left(j2\pi \times 3\times 10^6 \times n/N\right)\exp\left(-j2\pi \times 3\times 10^6 \times n/N\right)}^{N\,{}_3\mathbf{s}_m}$$

$$+ \overbrace{\sum_{n=0}^{N-1} {}_4\mathbf{s}_m \exp\left(j2\pi \times 4\times 10^6 \times n/N\right)\exp\left(-j2\pi \times 3\times 10^6 \times n/N\right)}^{0}$$

$$+ \overbrace{\sum_{n=0}^{N-1} {}_5\mathbf{s}_m \exp\left(j2\pi \times 5\times 10^6 \times n/N\right)\exp\left(-j2\pi \times 3\times 10^6 \times n/N\right)}^{0}$$

$$+ \overbrace{\sum_{n=0}^{N-1} {}_6\mathbf{s}_m \exp\left(j2\pi \times 6\times 10^6 \times n/N\right)\exp\left(-j2\pi \times 3\times 10^6 \times n/N\right)}^{0}$$

$$+ \overbrace{\sum_{n=0}^{N-1} {}_7\mathbf{s}_m \exp\left(j2\pi \times 7\times 10^6 \times n/N\right)\exp\left(-j2\pi \times 3\times 10^6 \times n/N\right)}^{0}$$

$$\left. + \overbrace{\sum_{n=0}^{N-1} {}_8\mathbf{s}_m \exp\left(j2\pi \times 8\times 10^6 \times n/N\right)\exp\left(-j2\pi \times 3\times 10^6 \times n/N\right)}^{0} \right]$$

$$= 0 + 0 + {}_3\mathbf{s}_m + 0 + 0 + 0 + 0 + 0 = {}_3\mathbf{s}_m \tag{1.6}$$

Dividing the third subcarrier by 2, and demonstrating that this act will violate the orthogonality and make demodulation of the symbol on that particular subcarrier unsuccessful, can be done in several way, i.e. at transmitter alone, at receiver alone and both at transmitter and receiver. Here we assume that the third subcarriers used at the transmitter are as those in (1.1) and (1.2), but the third subcarriers used at receiver are those third subcarriers of (1.1) and (1.2) divided by 2. Thus

$$f_3 = \frac{3}{T_{8\text{PSK}}} = 1 \text{ MHz}\ ,\ f_{3d8\text{PSK}} = \frac{f_3}{2} = 0.5 \text{ MHz}$$

$$f_3 = \frac{3}{T_{16\text{QAM}}} = \frac{3}{8} \text{ MHz}\ ,\ f_{3d16\text{QAM}} = \frac{f_3}{2} = \frac{3}{16} \text{ MHz} \qquad (1.7)$$

Because of the fractionality (in terms of MHz), it is best to test the above with analytic formulation, thus (1.4) will become

$$y(t) = \sum_{k=1}^{K} {}_k\mathbf{s}_m \exp\left(j2\pi f_k t\right) \qquad (1.8)$$

Then at receiver, demodulation on the third arm will generate the following result for 8 PSK and 16 QAM

$$d_{3d8\text{PSK}} = \int_0^T y(t)\ \exp\left(-j2\pi f_{3d8\text{PSK}} t\right) dt$$

$$= 1.9j\,{}_1\mathbf{s}_m + 1.9j\,{}_2\mathbf{s}_m + 0.64j\,{}_3\mathbf{s}_m + 0.38j\,{}_4\mathbf{s}_m + 0.27j\,{}_5\mathbf{s}_m + 0.21j\,{}_6\mathbf{s}_m + 0.17j\,{}_7\mathbf{s}_m + 0.15j\,{}_8\mathbf{s}_m \qquad (1.9)$$

$$d_{3d16\text{QAM}} = \int_0^T y(t)\ \exp\left(-j2\pi f_{3d16\text{QAM}} t\right) dt$$

$$= 5.1j\,{}_1\mathbf{s}_m + 5.1j\,{}_2\mathbf{s}_m + 1.7j\,{}_3\mathbf{s}_m + j\,{}_4\mathbf{s}_m + 0.73j\,{}_5\mathbf{s}_m + 0.57j\,{}_6\mathbf{s}_m + 0.46j\,{}_7\mathbf{s}_m + 0.39j\,{}_8\mathbf{s}_m$$

$$+ 0.34j\,{}_9\mathbf{s}_m + 0.3j\,{}_{10}\mathbf{s}_m + 0.27j\,{}_{11}\mathbf{s}_m + 0.24j\,{}_{12}\mathbf{s}_m + 0.22j\,{}_{13}\mathbf{s}_m + 0.2j\,{}_{14}\mathbf{s}_m + 0.19j\,{}_{15}\mathbf{s}_m + 0.17j\,{}_{16}\mathbf{s}_m \qquad (1.10)$$

As seen from (1.9) and (1.10), if the frequency of the third subcarrier is halved, demodulation for both 8 PSK and 16 QAM becomes unsuccessful. Note that the computations in (1.9) and (1.10) are done in the Matlab file, Calculations_Q1_FE_2013.m.

2. (35 Points) In a DS spread spectrum system, a 4 kbits/sec message signal is spread using $L_c = 1000$. Find and plot the following graphs,

   a) The spectrums of message signal prior to and after spreading,
   b) Time waveforms of message signal prior to and after spreading,

If an interference signal $I_i = 20\cos(20000\pi t) + 40\cos(2\times10^6\pi t)$ is mixed with received signal, find the signal power to interference power ratio (SIR) prior to demodulation (despreading) and after demodulation, assuming that the massage signal and the PN spreading sequence at transmitter have unity amplitude and the communication channel has a unity frequency response over the band of related frequencies.

Solution : The spectrums and the time waveform before and after spreading are shown in Figs 2.1 and 2.2

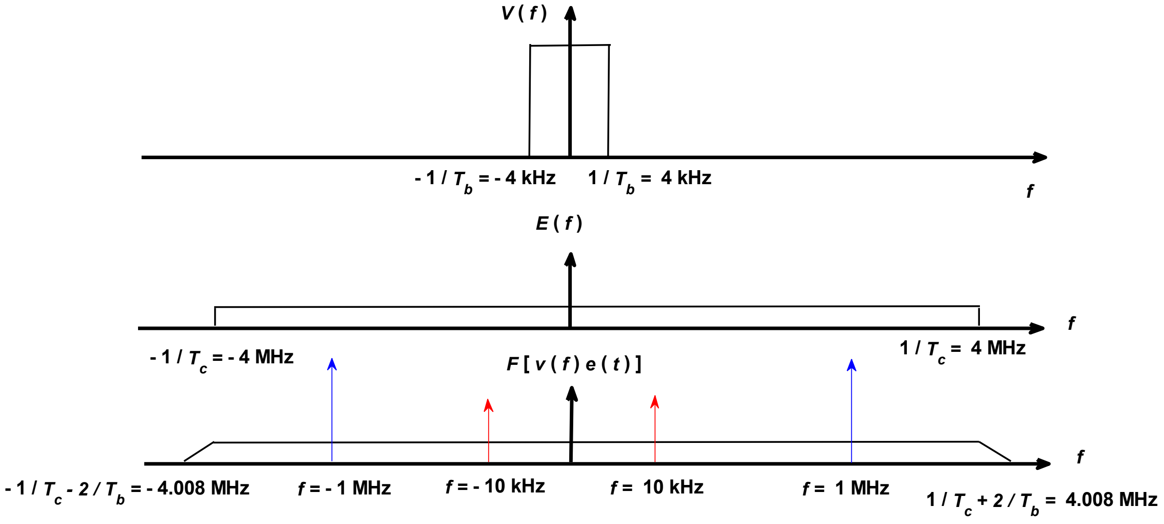

Fig. 2.1 The spectrums of the message signal before and after spreading, where on the last row, the interference signal is also included in the spectral range.

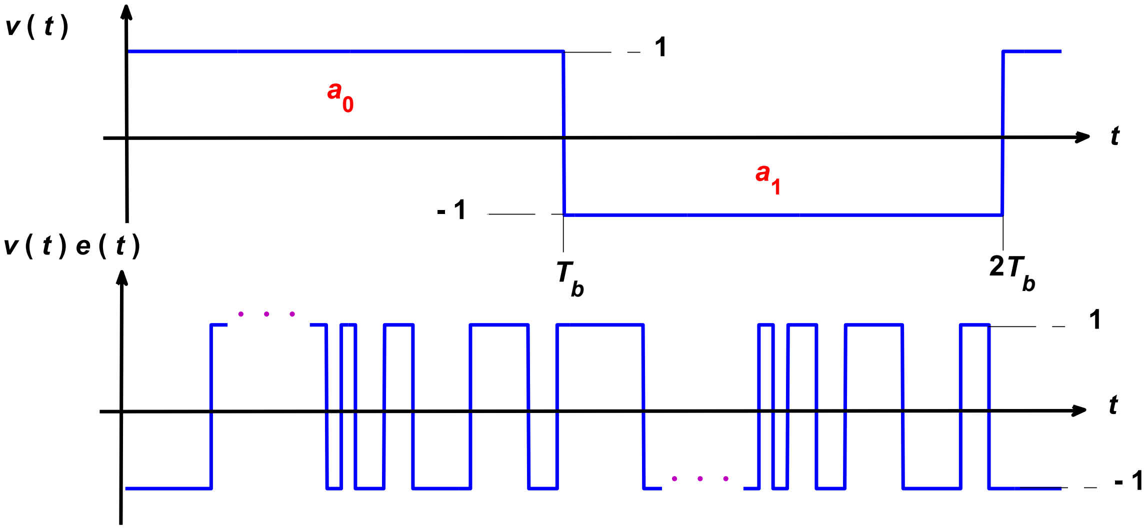

Fig. 2.2 Time waveforms of the message signal before and after spreading.

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Prior to demodulation

Signal power : $P_s = a_0^2 = 1 \text{ W}$, Interference power : $P_i = \left(20/\sqrt{2}\right)^2 + \left(40/\sqrt{2}\right)^2 = 1000 \text{ W}$

$$\text{SIR} = \frac{P_s}{P_i} = \frac{1}{1000} = 10^{-3} \quad \text{or} \quad -30 \text{ dB} \qquad (2.1)$$

After demodulation

Signal power : $P_s = a_0^2 = 1 \text{ W}$, Interference power : $P_i = 1000/L_c = 1 \text{ W}$

$$\text{SIR} = \frac{P_s}{P_i} = \frac{1}{1} = 1 \quad \text{or} \quad 0 \text{ dB} \qquad (2.2)$$

3. (30 Points) Answer the following questions as **True** or **False.** For the **False** ones give the correct answer or the reason. For the **True** ones justify your answer.

   a) TCM is based on PSK modulation : True, according to section 5 of ECE 587_Notes on Codes.

   b) TCM is based on QAM : True, according to section 5 of ECE 587_Notes on Codes.

   c) OFDM is used for communication channels with nonflat frequency responses : True, according to section 2 of Notes on OFDM_2013.

   d) When the message signal is spread from a narrow band into a wide band, its bandwidth remains the same : False, this is a spreading operation, thus the bandwidth of the message signal increases, as illustrated in Fig. 1.3 of Spread spectrum systems_2013_HTE.

   e) Maximum length PN sequences do not have any subsequence repetitions : True, as explained underneath of Eq. (3.4) of Spread spectrum systems_2013_HTE.